Time Value of Money Concepts

BASIC ANNUITIES
There are many accounting transactions that require the payment of a specific amount each period. A payment for a auto loan or a mortgage payment are examples of this type of transaction. If the interval between periods is constant and the amount is always the same, this is called an annuity. Because an annuity is a stream of payments over a period of time, our previous discussion of the time value of money would indicate that there must be an interest factor involved. In the language of annuities, the payments are called Rents (the symbol R is used to designate the amount of Rents). If the rent is paid at the end of each period the annuity is called an Ordinary Annuity. If the rent is paid at the beginning of each period the annuity is called an Annuity Due.

FUTURE VALUE OF AN ORDINARY ANNUITY
An example of an ordinary annuity is contributions to a 401-k retirement plan. At the end of each month a certain sum is deducted from the employee’s paycheck (or paid by the employer) and placed in an investment account.

The algebraic formula for the future value of an ordinary annuity is:

\[ FVOA = R \times [FVOA, n, i] \]

Let’s assume that Spencer Company has agreed to contribute $60,000 to an employee retirement plan at the end of each year. The investment yields a 12% annual return. If we assume that contributions will be made for 10 years then the following reflects the Future Value of the retirement plan contributions.

\[
FVOA = 60,000 \times [FVOA, n=10, i=12\%] \\
FVOA = 60,000 \times 17.54874 \\
FVOA = 1,052,924
\]

PRESENT VALUE OF AN ORDINARY ANNUITY
The algebraic formula for the present value of an ordinary annuity is:

\[ PVOA = R \times [PVOA, n, i] \]

In many cases business enterprises offer some kind of long-term benefits to employees after they retire. If Spencer Company has agreed to make semiannual contributions of $60,000 to a post-retirement medical insurance plan for 10 years and the investment vehicle earns 5% per annum what is the present value of this contribution schedule? Using the above formula we can calculate the present value of this ordinary annuity as follows:

\[
PVOA = 60,000 \times [PVOA, n=20, i=2.5\%] \\
PVOA = 60,000 \times 15.58916 \\
PVOA = 935,350
\]
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**FUTURE VALUE OF AN ANNUITY DUE**
As noted at the beginning of this lesson, an annuity due is one where the payments are made at the beginning of the period as opposed to the end of the period. This essentially adds one more period of compounding to the equation. A table is not available for the Future Value of an Annuity Due. To derive the appropriate factor, go to the Future Value of an Ordinary Annuity table and multiply the factor for the interest rate and period by 1 plus the periodic interest rate. For example, if Spencer Company agrees to make a $6,000 contribution to a defined benefit pension fund at the beginning of each quarter for three years and the fund earns 12% per annum then the following calculation would be used to determine the **Future Value of the Annuity Due**.

First we need to determine the factor for the future value of the annuity due as follows:

\[
\begin{align*}
\text{Future Value of Ordinary, } n=12, \ i=3\% & \quad 14.19023 \\
\text{Factor Adjustment, } 1+.03 & \quad x \quad 1.03 \\
\text{Future Value of Annuity Due, } n=12, \ i=3\% & \quad 14.61594
\end{align*}
\]

Then we can use this factor in our equation as follows:

\[
\begin{align*}
FVAD &= R \times [FVAD, n, i] \\
FVAD &= 6,000 \times [FVAD, n=12, \ i=3\%] \\
FVAD &= 6,000 \times 14.6159369 \\
FVAD &= 87,696
\end{align*}
\]

**PRESENT VALUE OF AN ANNUITY DUE**
In almost all lease situations the lessor (owner of the property) requires the lessee (user of the property) to make the rental payment at the beginning of the period. In financial accounting there are many occasions when we need to know the present value of the future lease payments. Because they are made at the beginning of the period these lease payments are annuity due payments. Again, we need to make the appropriate adjustment to the factor to determine the factor to be used in the Present Value of the Annuity Due. In this example, Spencer Company has agreed to rent to Salish Development a piece of commercial real estate. The lease agreement calls for rental payments of $500 per month payable at the beginning of each month for 3 years. The appropriate interest rate for this type of arrangement is 24% per annum. What is the present value of this annuity due lease agreement?

\[
\begin{align*}
\text{Present Value of Ordinary Annuity, } n=36, \ i=2\% & \quad 25.48884 \\
\text{Factor Adjustment, } 1+.02 & \quad 1.02 \\
\text{Present Value of Annuity Due, } n=36, \ i=2\% & \quad 25.99862
\end{align*}
\]

Now that we know the factor for the annuity due for 36 periods at 2% we can calculate the present value of lease.
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\[
\text{FVAD} = R \times \left[\text{FVAD, n, i}\right]
\]

\[
\text{FVAD} = 500 \times \left[\text{FVAD, n=36, i=2\%}\right]
\]

\[
\text{FVAD} = 500 \times 25.99862
\]

\[
\text{FVAD} = 12,999
\]

SOLVING FOR UNKNOWNS
As with single sum problems, there are situations when we know three of the four variables are known. The variables involved in annuity calculations are as follows:
1. The present (or future) value of the ordinary annuity (or annuity due),
2. The amount of the annuity payments,
3. The number of periods involved, and
4. The interest rate.

If you use the formula presented above we can solve any annuity problem.

Example: Spencer Company enters into a lease agreement for a piece of equipment. The fair value of the equipment is $149,500 and the terms of the lease are $50,000 per year for 5 years with each payment to be made at the end of the year. What is the effective interest rate that Spencer Company is paying for this piece of equipment? Would the company be better off purchasing the equipment if it’s cost of capital is 9% per annum?

Solution:

\[
\text{PVOA} = R \times \left[\text{PVOA, n=5, i=?}\right]
\]

\[
149,500 = 50,000 \times \left[\text{PVOA, n=5, i=?}\right]
\]

\[
\left[\text{PVOA, n=5, i=?}\right] = 149,500 / 50,000 = 2.99
\]

\[
i = 20\%
\]

If we know that the present value of the equipment is $149,500 and the five payments made at the end of each period are $50,000 we can calculate the table value of 2.99. If we look up in the present value of an ordinary annuity table for five years with a factor of 2.99 the interest rate is 20%.

DEFERRED ANNUITIES
Deferred annuities are annuities that start at some date other than the current period. Calculating the future value of an ordinary annuity involves tabulating the number of periods that rents will be made and using that as the number of periods. For example, Spencer Company is a little short on cash but would like to build a cash reserve within 10 years. The board of directors decides to start funding this cash reserve at the end of year 5 through year 10 at the rate of $10,000 per year. What will be the future value of the cash reserve fund if the annuity contributions earn 10% per annum?

<table>
<thead>
<tr>
<th>Period in years</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deferral period in years</td>
<td>4</td>
</tr>
</tbody>
</table>
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Period in years of annuity payments \(6\)

Therefore the future value of this ordinary annuity can be calculated as follows:

\[
\begin{align*}
\text{FVOA} & = R \times [FVOA, n, i] \\
\text{FVOA} & = 10,000 \times [FVOA, n=6, i=10\%] \\
\text{FVOA} & = 10,000 \times 7.71561 \\
\text{FVOA} & = 77,156
\end{align*}
\]

Calculating the present value of a deferred ordinary annuity is a little more complicated. In our example we did not start making payments until the end of year 5. Therefore if we use 6 periods and calculate the present value it will give us the present value at the end of year 5 as opposed to the beginning of year 1. Therefore we need to modify the factor used in order to end up with a factor that gives us the current present value. This is accomplished as follows:

\[
\begin{align*}
\text{Present Value of Ordinary Annuity, } n=10, i=10\% & = 6.14457 \\
\text{Subtract: } PVOA, n=4, i=10\% (\text{deferral}) & = 3.16986 \\
\text{Factor for PVOA deferred 4 periods} & = 2.97471
\end{align*}
\]

The factor that will be used is 2.97471, so we are now ready to calculate the present value of the ordinary annuity deferred for four periods.

\[
\begin{align*}
\text{PVOA} & = R \times [PVOA, n, i] \\
\text{PVOA} & = 10,000 \times [PVOA, n=6, i=10\%] \\
\text{PVOA} & = 10,000 \times 2.97471 \\
\text{PVOA} & = 29,747
\end{align*}
\]

**VALUATION OF LONG-TERM BONDS**

Chapter 14 covers the issuance and retirement of long-term bonds. You will use present value calculations in determining the issue price and the gains or losses on early extinguishment of debt. Bonds reflect two distinctly separate cash flows: the principal and the periodic interest payments. To calculate the issue price of bonds we first calculate the present value of the principal payment that will be made in the future. We then calculate the present value of the periodic interest payments (ordinary annuity). We then add these two amounts together to determine the issue price of the bonds.

For example, Spencer Company issues $1,000,000 of 10% bonds due in 20 years. Interest is paid semiannually on June 30 and December 31. The current market rate of interest when the bonds are issued is 8%. Therefore the bonds are issued at a premium. In other words the buyers of the bonds only need to yield 8% to be satisfied so they will pay more than the face value for the bonds. Our job is to calculate the issue price so that we can book the issuance of the bonds along with the appropriate premium.

This is accomplished in two steps. In step #1 we will calculate the present value of the principal payment in 20 years using the market interest rate. In step #2 we will calculate the interest
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payments attached to the bonds. This is our ordinary annuity. Then we will discount this ordinary annuity for 20 years or 40 periods at the current market interest rate to determine the present value of this ordinary annuity.

**Present Value of Principal**

<table>
<thead>
<tr>
<th>Principal</th>
<th>1,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV$1, n=40, i=4%</td>
<td>0.20829</td>
</tr>
<tr>
<td>Present Value of Principal</td>
<td>208,290</td>
</tr>
</tbody>
</table>

**Present Value of Periodic Interest Payments**

<table>
<thead>
<tr>
<th>Principal</th>
<th>1,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stated Interest Rate per period</td>
<td>5%</td>
</tr>
<tr>
<td>Interest per period (ordinary annuity)</td>
<td>50,000</td>
</tr>
<tr>
<td>PVOA, n=40, i=4%</td>
<td>19.79277</td>
</tr>
<tr>
<td>Present Value of Ordinary Annuity</td>
<td>989,639</td>
</tr>
</tbody>
</table>

**Issue Price of Bonds**

<table>
<thead>
<tr>
<th></th>
<th>1,197,929</th>
</tr>
</thead>
</table>

The bonds have a face value of $1,000,000 so the journal entry to record the bonds in Spencer Company’s books will be as follows:

<table>
<thead>
<tr>
<th>ACCOUNT</th>
<th>DEBIT</th>
<th>CREDIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>1,197,929</td>
<td></td>
</tr>
<tr>
<td>Bonds Payable</td>
<td>1,000,000</td>
<td></td>
</tr>
<tr>
<td>Premium on Bonds Payable</td>
<td>197,929</td>
<td></td>
</tr>
</tbody>
</table>